The causal effects of modified treatment policies under network interference

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Scientific Motivation: Environmental Health

Example domains

- Air pollution
- Wildfires
- Extreme heat

Common issue: *continous treatments*

Standard causal data set-up

Observed data: A tuple of n-vectors, $O_1, \ldots, O_n$, where

$$\mathbf{O} = (\mathbf{L}, \mathbf{A}, \mathbf{Y}) \sim \mathsf{P}$$

- $\mathbf{L}$: measured baseline covariates
- $\mathbf{A}$: continuous exposure
- $\mathbf{Y}$: outcome of interest

Question: how much would Y have changed under different value of A?

Causal inference with continuous A

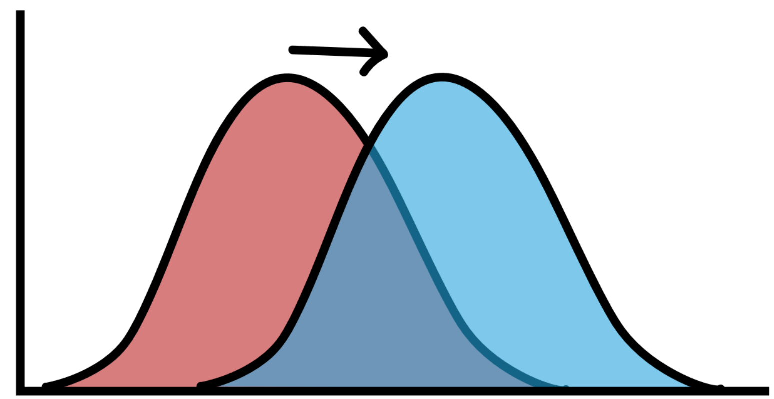
- Let $Y(a)$ denote "potential outcome": value of Y had we set $A = a$.
- Typically seek counterfactual mean $E(Y(a))$
  - → average effect on Y of setting $A = a$
- If A is continuous...
  - → Can't observe all possible A: hard to estimate dose-response nonparametrically
  - → "Setting all $A = a$" often doesn't make sense
  - → Instead, consider *modifying* observed treatment...

Modified Treatment Policies

A user-specified function $\boldsymbol{d(A, L; \delta)}$ that maps the observed exposure $\boldsymbol{A}$ to an post-intervention value $\boldsymbol{A^d}$ (Haneuse and Rotnitzky 2013).

- *Additive*: $\boldsymbol{d(A, L; \delta) = A + \delta}$
- *Multiplicative*: $\boldsymbol{d(A, L; \delta) = \delta \cdot A}$
- *Piecewise Additive*:

$$d(A, L; \delta) = \begin{cases} A + \delta \cdot L & A \in \mathcal{A}(L) \\ A & \textbf{otherwise} \end{cases}$$



Causal Effect of a Modified Treatment Policy

Counterfactual mean is now

$$\mathbb{E}_{\mathsf{P}}\Big(Y(d(A, L; \delta))\Big) = \mathbb{E}_{\mathsf{P}}\Big(Y(A^d)\Big)$$

and *population intervention effect* is $E(Y(A^d)) - E(Y)$

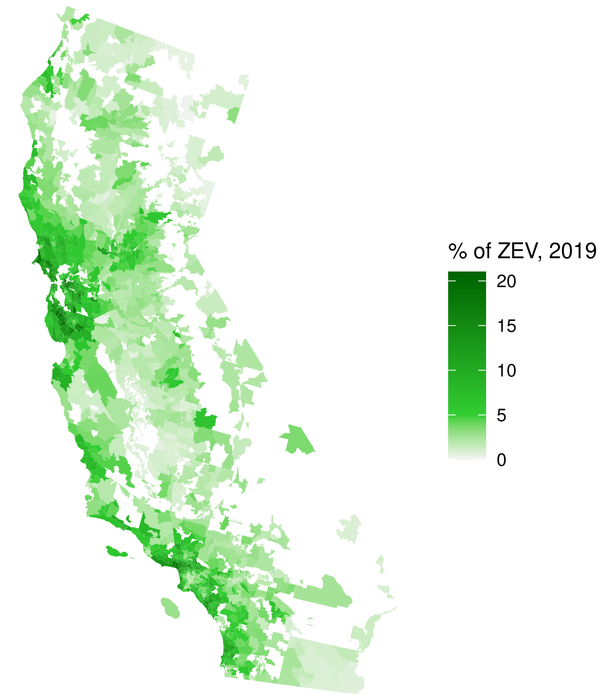
- "Average Y caused by shifting each A_i by d"
- Causal, nonparametric analogue of a linear regression coefficient

Problem: want MTP effects in **spatial data**...

Example: Electric Vehicles

What is the impact of zero-emissions vehicles (ZEV) on NO_2 air pollution in California?

- Continuous treatment (proportion of ZEVs)
- No real-world intervention can "set everyone's proportion of ZEVs to $A = a$"
- But we can consider MTP effects, like $E(Y(A+1))$ or $E(Y(1.01 \cdot A))$



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Research Question

How to **identify** and **estimate** causal effects of MTPs in spatial data?

Must be...

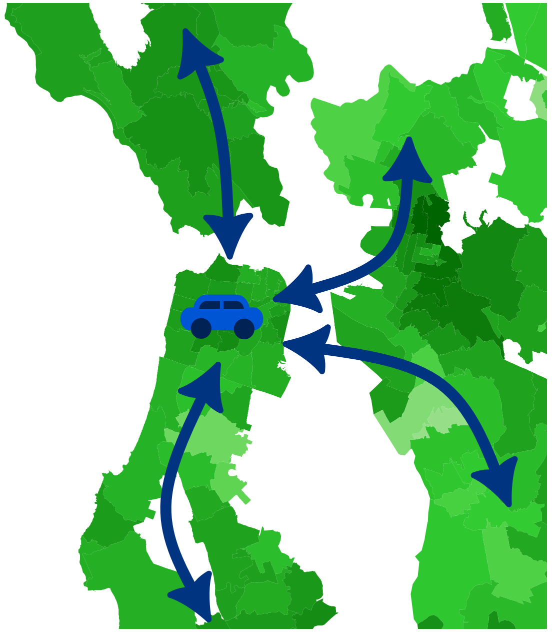
- Policy-relevant (*intervention on population*)
- Flexibly estimable (*no parametric nuisance models*)
- Efficient (*approach lowest possible variance*)

Interference

Hudgens and Halloran (2008): *interference* occurs when potential outcome of unit $\boldsymbol{i}$ depends on exposures of *other units*

$$Y_i(a_i, a_j) \neq Y_i(a_i, a_j') \text{ if } a_j \neq a_j'$$

- Common in spatial data
- Causal identification fails: SUTVA/consistency violated
- Correlated data → challenging estimation



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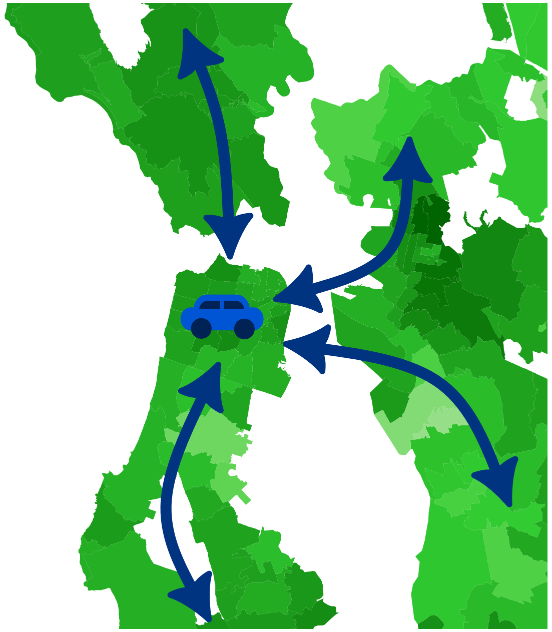
Network interference: Potential outcomes only depend on neighbors in adjacency matrix $\mathbf{F}$ (van der Laan 2014).



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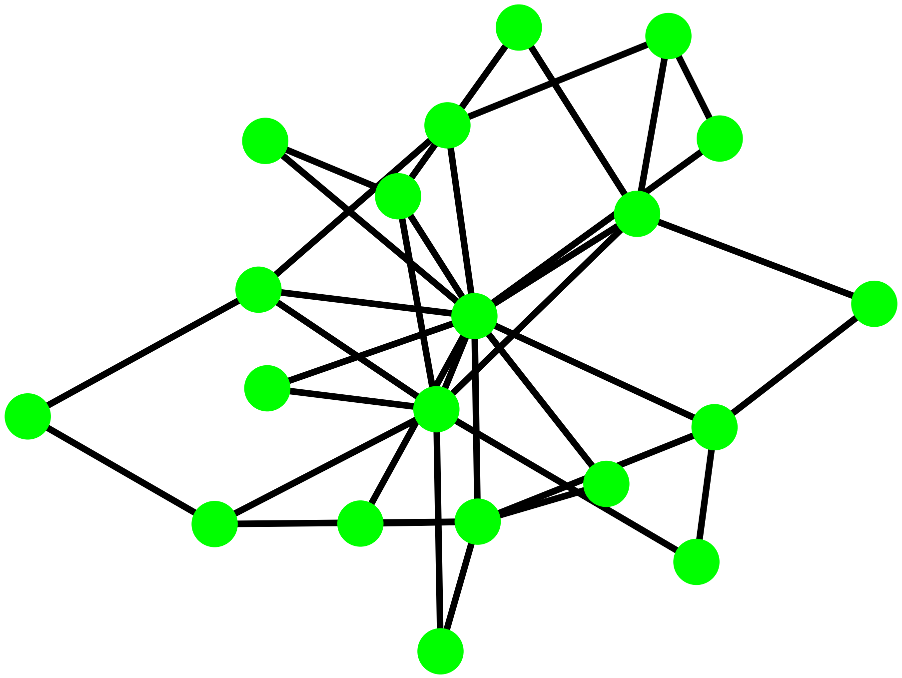
The Induced MTP

New Data Structure

1. **Observed data**: A tuple of n-vectors, $O_1, \ldots, O_n$, where

$$\mathbf{O} = (\mathbf{L}, \mathbf{A}, \mathbf{Y})$$

2. **Network $\mathbf{F}$**: An adjacency matrix of each unit's neighbors (known).



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Repairing identification under interference

Under interference, consider the following structural equation:

$$Y_i = f\Big(\boldsymbol{s}_A(A_j : j \in F_i), \boldsymbol{s}_L(L_j : j \in \mathbf{F}_i)\Big)$$

- $\boldsymbol{s}$: "summary" of neighbors' exposures or **exposure mapping**
- For short, denote vector of $\boldsymbol{s}_A(A_j : j \in \mathbf{F}_i)$ as $\boldsymbol{s}(A)$
- Example: $\boldsymbol{s}(A)_i = \sum_{j \in \mathcal{F}_i} A_j$

Treating $\boldsymbol{s}(A)$ as exposure instead of A restores SUTVA (Aronow and Samii 2017); just use $Y(\boldsymbol{s}(a))$ instead of $Y(a)$!

The induced MTP

What happens if we apply the MTP *and then* summarize?

$$A \xrightarrow{d} A^{d} \xrightarrow{s} A^{s \circ d}$$

Call the function $s \circ d$ the **induced MTP**.

Population intervention effect of an induced MTP:

$$\Psi_n(\mathsf{P}) = \mathbb{E}_{\mathsf{P}}\left[\frac{1}{n}\sum_{i=1}^{n} Y_i(s(d(\mathbf{A}, \mathbf{L}; \delta))_i)\right] - \mathbb{E}_{\mathsf{P}}\left[Y\right]$$

- *data-adaptive*, only observe one network

Identification

Network analogues of classical assumptions (weaker)

A0 (SCM). Data are generated from a structural causal model:

$$L_i = f_L(\varepsilon_{L_i}); A_i = f_A(L_i^s, \varepsilon_{A_i}); Y_i = f_Y(A_i^s, L_i^s, \varepsilon_{Y_i}) \ .$$

with error vectors independent of each other with identically distributed entries and $\varepsilon_i \perp\!\!\!\perp \varepsilon_j$ provided i, j not neighbors in $\mathbf{F}$

A1 (Summary positivity). If $s(a), s(l) \in \mathbf{supp}(A^s, L^s)$ then $s(a^d), s(l) \in \mathbf{supp}(A^s, L^s)$

A2 (No unmeasured confounding). $Y(A^s) \perp\!\!\!\perp A^s \mid L$

Extra *necessary* conditions on $\boldsymbol{d}$ and $\boldsymbol{s}$

A3 (Piecewise smooth invertibility). The MTP $\boldsymbol{d}$ has an differentiable inverse on a countable partition of $\mathbf{supp}(\boldsymbol{A})$.

A4 (Summary coarea). $\boldsymbol{s}$ has Jacobian $J\boldsymbol{s}$ satisfying

$$\sqrt{\det J\boldsymbol{s}(\boldsymbol{a}) J\boldsymbol{s}(\boldsymbol{a})^\top} > 0$$

- From measure-theoretic calculus; allows use of $A^{\boldsymbol{s}}$ in place of $\mathbf{A}$

Identification Result (Section S2)

Statistical estimand factorizes in terms of $\boldsymbol{A}^s$:

$$\psi_n = \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}_{\mathsf{P}}\left(m(A_i^s, L_i^s) \cdot r(A_i^s, A_i^{s\circ d}, L_i^s) \cdot w(\mathbf{A}, \mathbf{L})_i\right)$$

with nuisance parameters $\boldsymbol{m}$, $\boldsymbol{r}$ and weights $\boldsymbol{w}$:

$$m(a^s, l^s) = \mathbb{E}_Y(Y \mid A_i^s = a^s, L_i^s = l^s)$$

$$r(a^s, a^{s\circ d^{-1}}, l^s) = \frac{p(a^{s\circ d^{-1}} \mid l^s)}{p(a^s \mid l^s)}$$

$$w(\mathbf{a}, \mathbf{l}) = \sqrt{\frac{\det J(s\circ d^{-1})(\mathbf{a}) J(s\circ d^{-1})(\mathbf{a})^\top}{\det Js(\mathbf{a}) Js(\mathbf{a})^\top}}$$

Advantages of MTP in network

- **Population-level** estimand; intervention always compatible with network
- Fewer **positivity issues** from enforcing static intervention on summaries themselves
- Unknown parts of estimand only in terms of A^s and L^s

Estimation

Desiderata for estimators

- *semiparametric efficiency*
  - → Best possible variance among the class of regular asymptotically linear (RAL) estimators
- *rate double-robustness*
  - → structure allows flexible regression or machine learning (converge slower than $o_{\mathbb{P}}(n^{-1/2})$, parametric rate) for nuisance estimation

Efficient, Doubly-Robust, Nonparametric Estimation

Construct an efficient estimator solving estimating equation with **efficient influence function ϕ**

$$\frac{1}{n}\sum_{i=1}^{n}\phi(O_i;\hat{\eta})$$

where $\hat{\eta}$ is a set of nuisance estimators whose *product* converge at $o_{\mathbb{P}}(n^{-1/2})$ (i.e. only need $o_{\mathbb{P}}(n^{-1/4})$), typical in statistical learning)

- One-step correction (Bickel et al. 1993; Pfanzagl and Wefelmeyer 1985) (e.g. AIPW)
- TMLE (van der Laan and Rose 2011; van der Laan and Rubin 2006)

Efficient, Doubly-Robust, Nonparametric Estimation

The efficient influence function of ψ_n, a special case of the EIF for the counterfactual mean of a stochastic intervention (Ogburn et al. 2022), is

$$\bar{\phi}(O_i) = \frac{1}{n}\sum_{i=1}^{n} w(\mathbf{A}, \mathbf{L})_i \cdot r(A_i^s, L_{s,i})(Y_i - m(A_i^s, L_i^s)) \\ + \mathbb{E}(m(A_i^{s \circ d}, L_i^s; \delta), L_i^s) \mid \mathbf{L} = \mathbf{l}) - \psi_n \ ,$$

Efficient, Doubly-Robust, Nonparametric Estimation

Ogburn et al. (2022)'s CLT: If $\hat{\psi}_n$ is constructed to solve $\bar{\phi} \approx \mathbf{0}$ and $K^2_{\max}/n \to 0$, then, under mild regularity conditions,

$$\sqrt{C_n}(\hat{\psi}_n - \psi_n) \to \mathrm{N}(0, \sigma^2) \ ,$$

where $K_{\max}$ is the network's maximum degree.

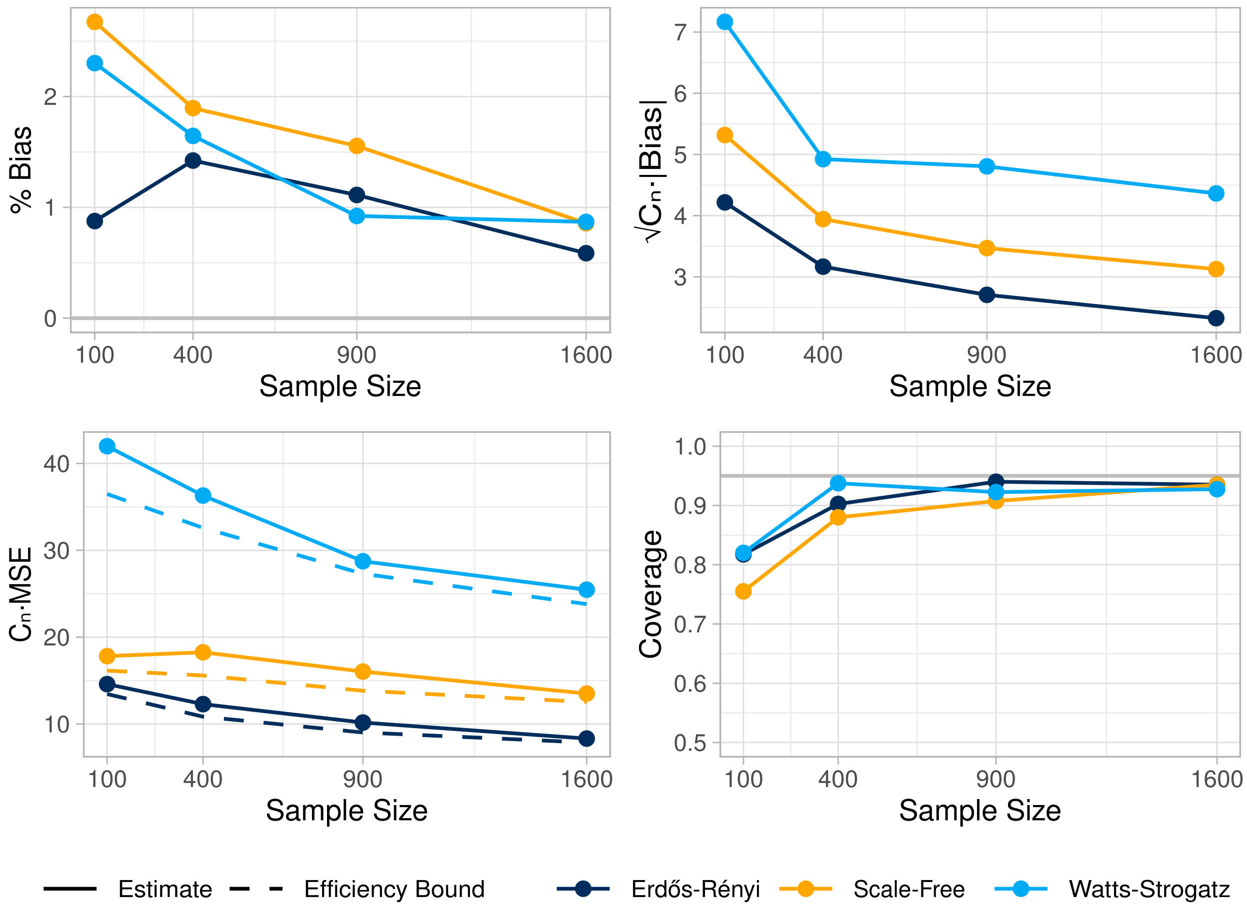
The estimator $\hat{\psi}_n$ is asymptotically normal, but the rate depends on a factor $n/K^2_{\max} < C_n < n$ (automatically contained within $\hat{\sigma}^2$)

Estimation Framework

1. Fit estimators $\hat{m}$ and $\hat{r}$ of nuisance parameters m and r via cross-fitting[1] and super (ensemble machine) learning (Davies and van der Laan 2016; van der Laan et al. 2007).
2. Construct one-step or "network-TMLE" estimators (Zivich et al. 2022) from an estimated EIF based on $\hat{m}$ and $w \cdot \hat{r}$ (weighted density ratio)
3. Compute standard error and construct Wald-style confidence intervals based on empirical variance of the estimated EIF[2].

Empirical results

Asymptotic properties of Network-TMLE



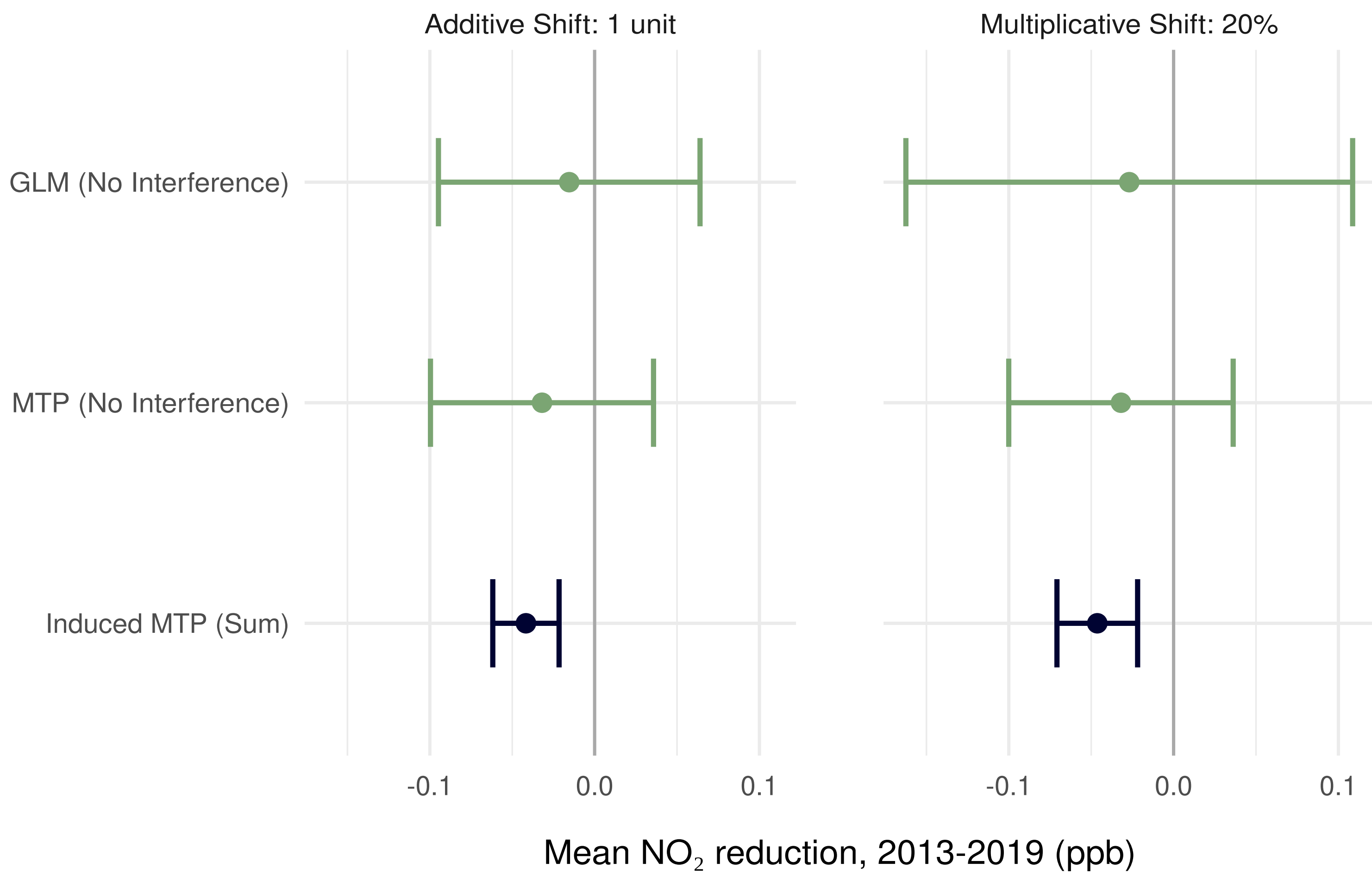
Versus competing methods on semisynethic data

- Simulate A and Y as linear models from 16 socioeconomic and land-use ZIP-code level covariates from the ZEV-NO_2 California dataset.
- *How poor would estimates be if only mistake were ignoring interference?*

Method	Learner	% Bias	Variance	Coverage
Network-TMLE	Correct GLM	0.11	1.56	96.2%
Network-TMLE	Super Learner	1.03	1.56	94.0%
IID-TMLE	Correct GLM	20.42	2.11	54.8%
Linear Regression	—	20.62	2.12	55.0%

Data Analysis

Effect of electric vehicles on $\mathbf{NO_2}$ in California



- GLM (ignores interference): **ZEVs reduce NO_2 by 0.015 ppb**, totaling ~2.5% of average change in NO_2
- Induced MTP: **ZEVs reduce NO_2 by 0.042 ppb**, totaling ~7% of average change in NO_2

Further Work

Further work

Challenges remain:

- Difficult to estimate conditional density ratio nuisance $\boldsymbol{r}$
  - → May benefit from undersmoothing or "Riesz learning"
- If summaries $\boldsymbol{s}$ unknown, can we learn them automatically?
- Same theory of the Longitudinal MTP (Díaz et al. 2021) should extend when reduced; useful for time-varying setting

Simulations powered by CausalTables.jl

The Journal of Open Source Software

CausalTables.jl: Simulating and storing data for statistical causal inference in Julia

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Software

- Review
- Repository
- Archive

Editor: Oskar Laverny

Reviewers:

Summary

Estimating the strength of causal relationships between treatment and response variables is an important problem across many scientific disciplines. `CausalTables.jl` is a package that supports causal inference in Julia by providing two important functionalities. First, it implements the `CausalTable`, bundling tabular data with a type of directed acyclic graph (DAG) encoding features' causes. Users can intervene on treatments and identify causal-relevant variables like confounders automatically. Second, the package's `StructuralCausalModel` interface simplifies

Thank you! Questions?

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Appendix A: Variance estimation

EIF was given in the form $\frac{1}{n}\sum_{i=1}^{n}\phi_P(O_i)$, but must be centered at the means of units with the same number of neighbors $N(|\mathbf{F}_i|)$:

$$\varphi_i = \phi_{\hat{P}_n(O_j)}(O_i) - \frac{1}{|N(|\mathbf{F}_i)|)|}\sum_{j\in N(|F_i|)}\phi_{\hat{P}_n(O_j)}$$

Then, $\hat{\sigma}^2 = \frac{1}{n^2}\sum_{i,j}\mathbf{F}_{ij}\varphi_i\varphi_j \xrightarrow{P} \sigma^2$

Appendix B: Cross-fitting in Dependent Data

Main idea: cross-fitting eliminates the "empirical process term"

$$\mathbb{P}_n\phi_{\hat{\eta}} = \underbrace{\mathbb{P}_n\phi_{\eta_0}}_{\textbf{CLT}} + \underbrace{\mathbb{P}(\phi_{\hat{\eta}} - \phi_{\eta_0})}_{\textbf{Nuisance product}} + \underbrace{(\mathbb{P}_n - \mathbb{P})(\phi_{\hat{\eta}} - \phi_{\eta_0})}_{\textbf{Empirical process}}$$

- Empirical mean unbiased under cross-fitting, even in correlated units
- $\mathbf{Var}(\phi_{\hat{\eta}} - \phi_{\eta_0}) = o(1/C_n)$ by Bienayme's identity
  - → Network assumes $K_{\max}^2/n \leq C_n$
  - → There are at most $K_{\max}^2$ correlated units
- Therefore, $(\mathbb{P}_n - \mathbb{P})(\phi_{\hat{\eta}} - \phi_{\eta_0}) = o_P(1/C_n)$

Appendix C: DGP for simulation study

Draw 400 iterations, estimate effect of MTP based on

$$\mathbf{L}_1 \sim \text{Beta}(3,2); \mathbf{L}_2 \sim \text{Poisson}(100); \mathbf{L}_3 \sim \text{Gamma}(2,4); \mathbf{L}_4 \sim \text{Bernoulli}(0.6)$$

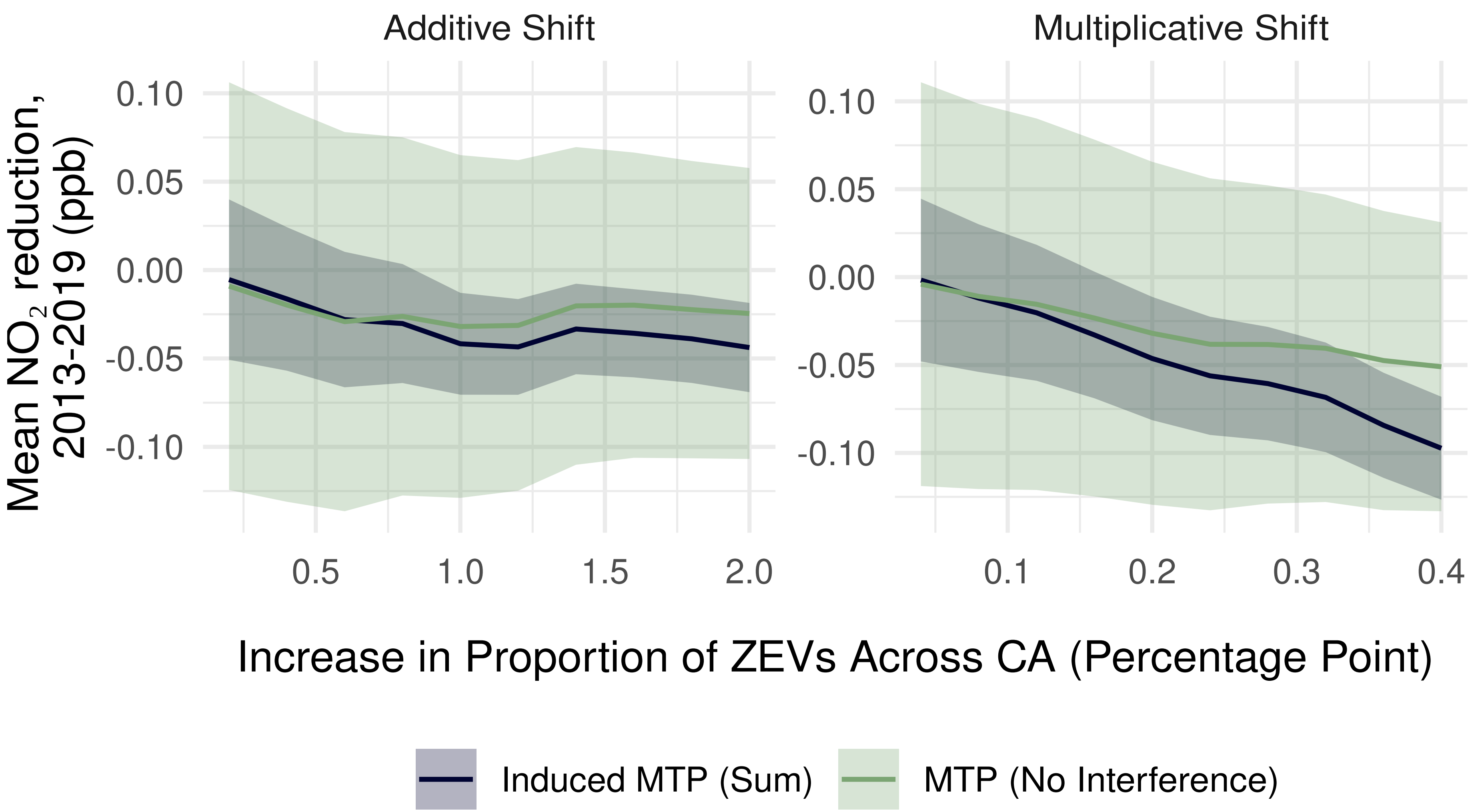
$$m_L = \Big(1 + L_4\Big) \cdot \Big(-2(\mathbb{I}(L_1 > 0.3) + \mathbb{I}(L_2 > 90) + \mathbb{I}(L_3 > 5)) - (\mathbb{I}(L_1 > 0.5) + \mathbb{I}(L_2 > 100) + \mathbb{I}(L_3 > 10)) + 2(\mathbb{I}(L_1 > 0.7) + \mathbb{I}(L_2 > 110) + \mathbb{I}(L_3 > 15))\Big)$$

$$\mathbf{A} \sim \text{Normal}(m_L - 5, 1.0) \text{ and } \mathbf{A}^s = \Big[\sum_{j \in F_i} A_i\Big]_{i=1}^{n}$$

$$m_A = -2\mathbb{I}(A > -2) - \mathbb{I}(A > 1) + 3\mathbb{I}(A > 3); m_{A_s} = 3\mathbb{I}(A_s > 0) + \mathbb{I}(A_s > 6) + \mathbb{I}(A_s > 12)$$

$$\mathbf{Y} \sim \text{TruncNormal}(m_L \cdot (1 + 0.2 m_A + m_{A_s}) + 5, 2.0) \ ,$$

Appendix D: Effect of ZEV on $\mathbf{NO_2}$



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