Nonparametric Network Causal Inference for Continuous Exposures in Mobile Source Air Pollution

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Motivation

Observational studies with data $O = (W, A, Y)$ in environmental health often seek to estimate a causal effect in a setting where...

1. The exposure A is **continuous-valued** + 2. The treatment of one unit affects the outcome of others (**interference**)

How can we perform machine-learning-based causal inference with continuous exposures and arbitrary network interference?

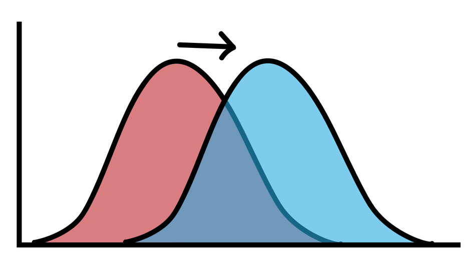
Modified Treatment Policy

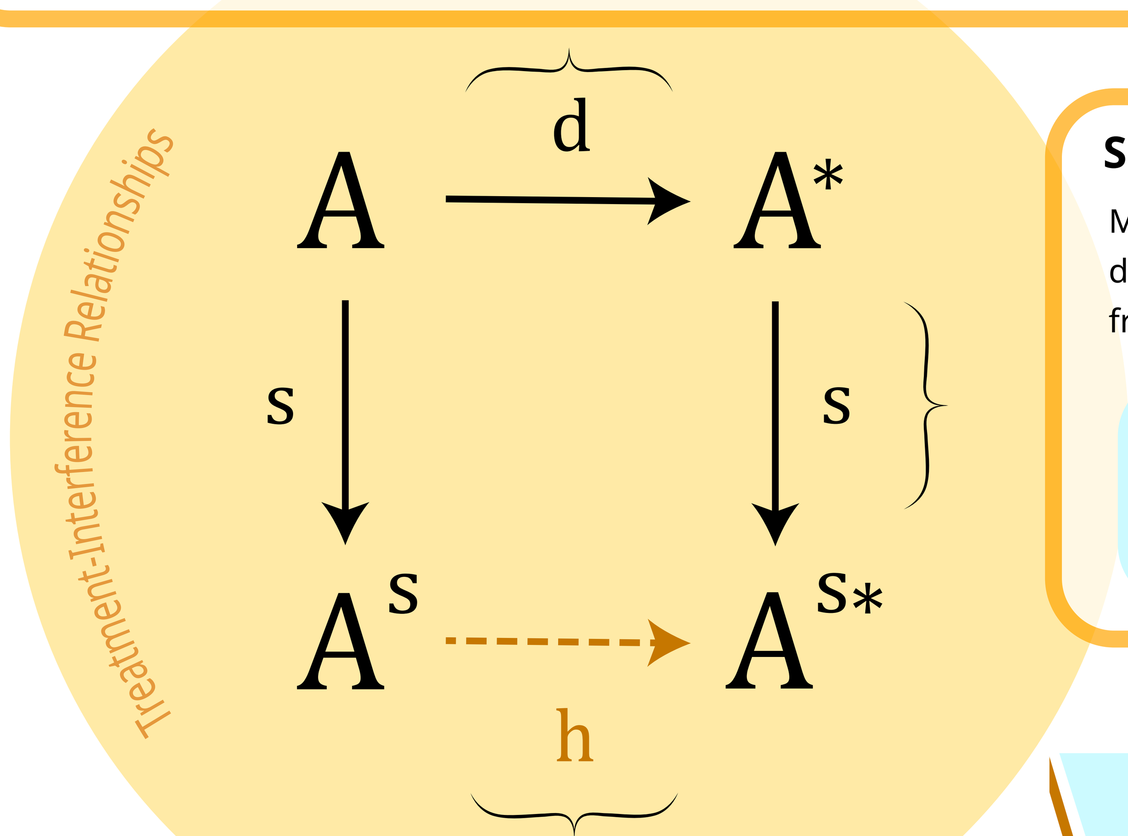
(Haneuse and Rotnizky, 2013; Diaz et al. 2018, 2021)

A causal estimand that addresses continuous-valued treatments by representing the counterfactual where some function d is applied to the natural value of exposure across all units.

Example (Additive Shift): $d(a_i, w_i; \delta) = a_i + \delta(w_i)$

Example (Multiplicative Shift): $d(a_i, w_i; \delta) = \delta(w_i) \cdot a_i$



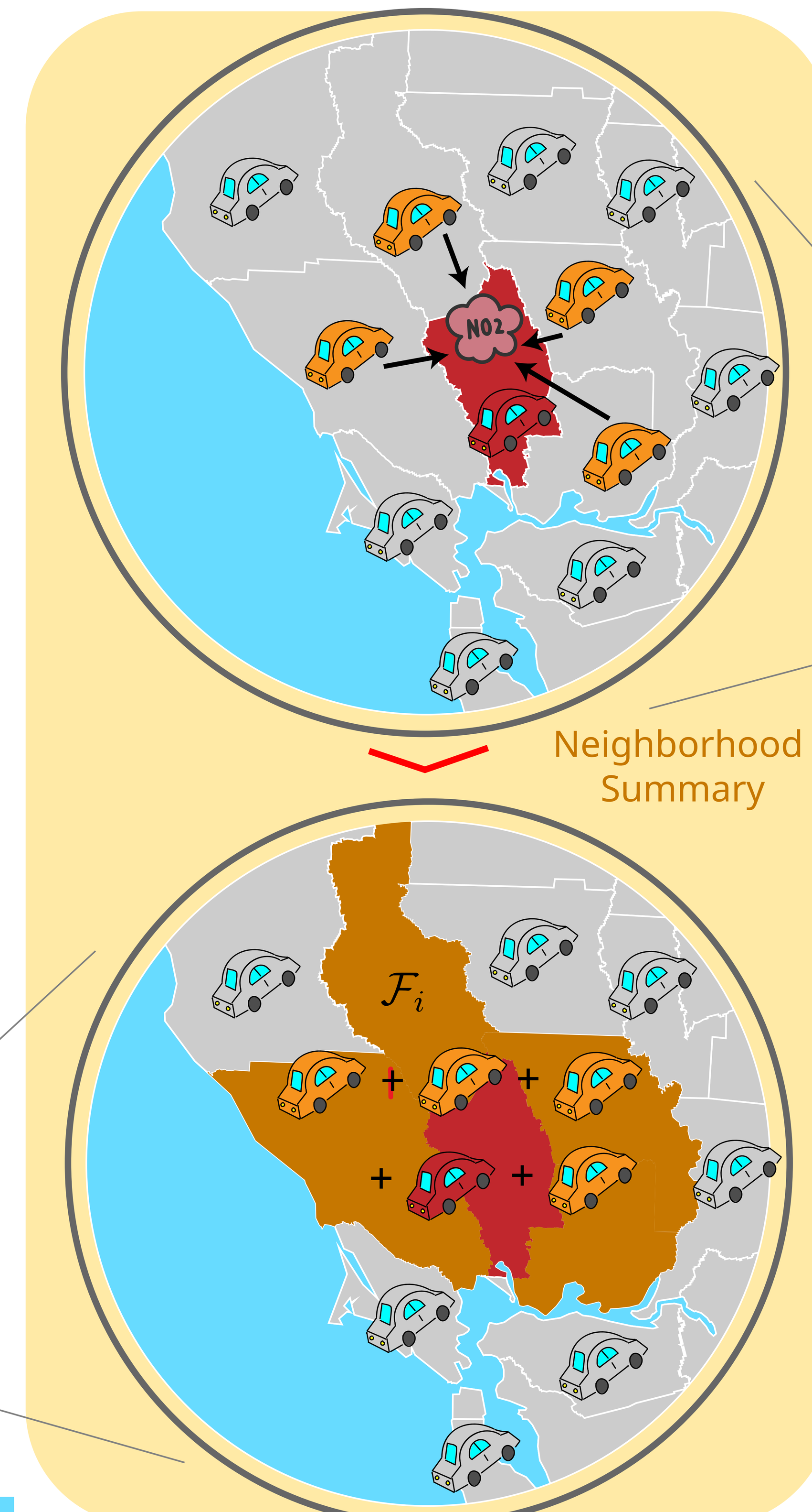


Summary Function:

Measures interference; a symmetric function describing the total exposure *received* by unit i from the set of its neighbors $\mathcal{F}_i$

Example (Weighted Sum of Neighbors):

$$s(i, \mathbf{a}, \mathbf{w}) = \sum_{j \in \mathcal{F}_i} w_j a_j$$



Induced Modified Treatment Policy:

A function h satisfying $(h \circ s)(a_i, w_i; \delta) = (s \circ d)(a_i, w_i; \delta)$ that is piecewise invertible (required to form a valid MTP)

Q: When does such a function exist?

Identifying Assumptions

(1) **Consistency:** $A^s = a^s \implies Y^{a^s} = Y$

(2) **Positivity:** $(a^s, w^s) \in \text{supp}(A^s, W^s) \implies (a^{s*}, w^s) \in \text{supp}(A^s, W^s)$

(3) **Conditional exchangeability*:** $Y^{h(a^s, \mathbf{w}; \delta)} \perp\!\!\!\perp A^s | W^s$ (*may be relaxed)

(4) **Piecewise smooth invertibility:** h has a piecewise differentiable inverse

Under (1)-(4), the Induced MTP is identified

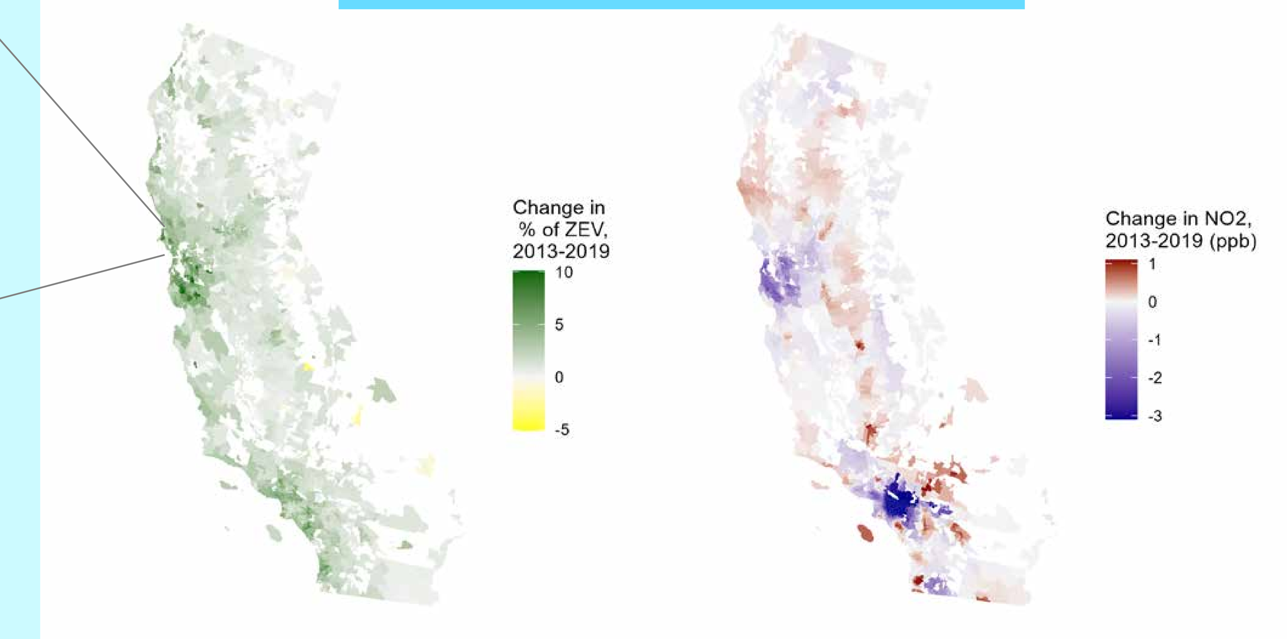
$$\psi = E(Y^{h(a^s, \mathbf{w}; \delta)}) = \int E(Y | A^s = h(a^s, \mathbf{w}; \delta), W^s) dP(A^s, W^s)$$

and can be efficiently estimated nonparametrically.

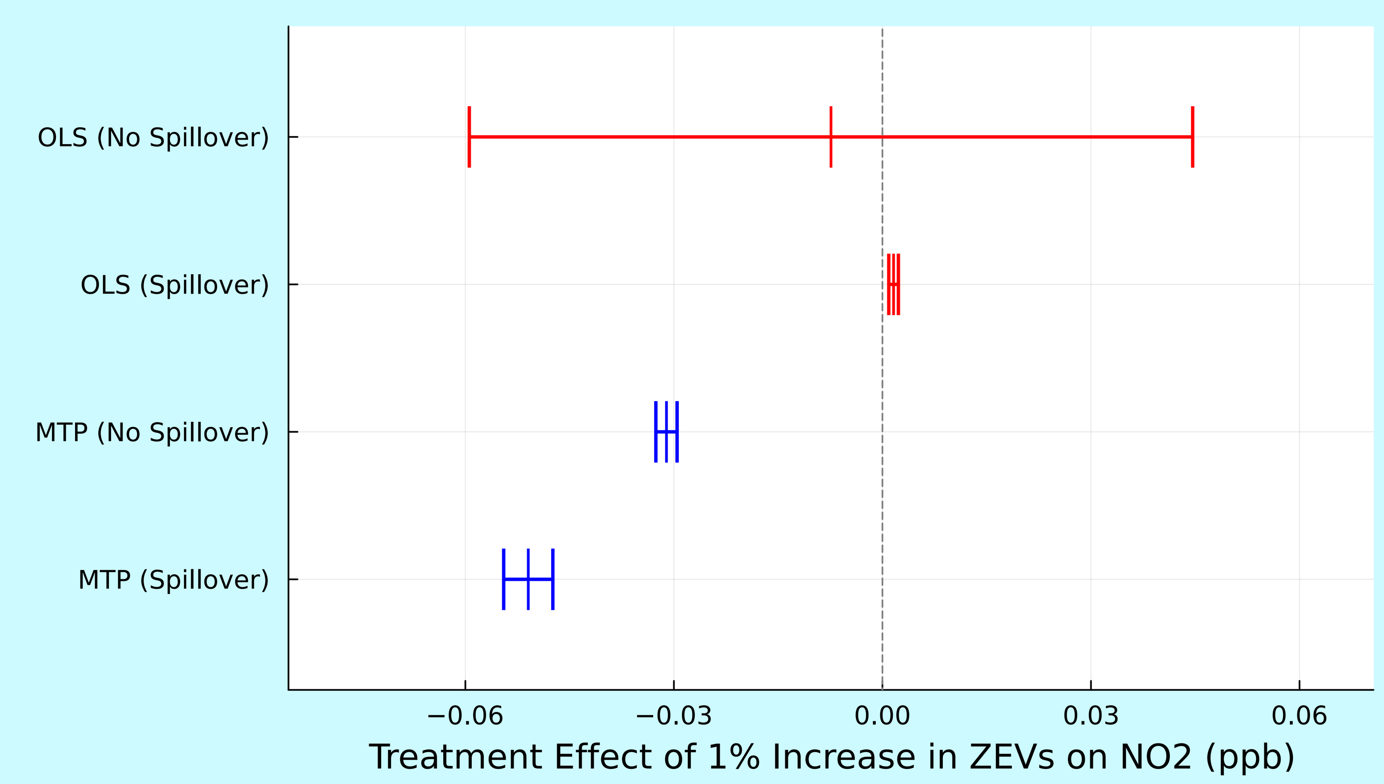
Data Application

Evaluating the effect of "zero-emission vehicle" (ZEV) uptake on NO2 pollution in California (unit of analysis: ZIP Code Tabulation Area)

Change in ZEV and NO2 across California



- Controlled for socioeconomic factors, road density, walkability, industrial zoning, and transportation use variables.
- MTP Estimation: Super Learning (*van der Laan et al. 2007*) with ElasticNet, Random Forest, and LGBM for outcome regression, KLIEP (*Sugiyama et al. 2007*) for density ratio.
- Covariate-conditional variance reported.



Theoretical Results

Definition (Selector): s is a selector if $s(i, \mathbf{a}, \mathbf{w}) = f(a_j, w_j)$ for some $a_j \in \mathcal{F}_i$ and f is piecewise smooth invertible.

Example (Maximum): $s(i, \mathbf{a}, \mathbf{w}) = \max\{a_j \in \mathcal{F}_i\}$

Definition (Aggregator): If $(\mathcal{A}, +, \cdot)$ is a field, and f, g field homomorphisms, s is an aggregator if $s(i, \mathbf{a}, \mathbf{w}) = g\Big(\sum_{j \in \mathcal{F}_i} f(a_j, w_j)\Big)$

Example (Weighted Mean): $s(i, \mathbf{a}, \mathbf{w}) = \frac{1}{|\mathcal{F}_i|} \sum_{j \in \mathcal{F}_i} w_j a_j$

Theorem 1

An induced modified treatment policy is piecewise smooth invertible (satisfying assumption 4) if either:

1. s is a selector **OR**
2. s is an aggregator (or a composition of them) with differentiable operations and the unit-level MTP takes the form

$$d(a_i, w_i; \delta) = \sum_j \Big(\delta \cdot a_i + q(w_i, \delta)\Big) \cdot 1(a_i \in \mathcal{A}_j)$$

Estimation

Nuisance Parameters

Outcome Regression: $\bar{Q}_i(a_i^s, w_i^s) = E(Y | A^s = a_i^s, W^s = w_i^s)$

Density Ratio: $H_i = \frac{p(h^{-1}(a_i^s, \mathbf{w}; \delta) | w_i^s)}{p(a_i^s | w_i^s)} \cdot \frac{d}{da^s} h^{-1}(a^s, \mathbf{w}; \delta)\Big|_{a^s = a_i^s}$

Estimators

Plug-In: $\hat{\psi}_n = \frac{1}{n} \sum_{i=1}^n \bar{Q}_i(a_i^s, w_i^s)$

IPW: $\hat{\psi}_n = \frac{1}{n} \sum_{i=1}^n H_i Y_i$

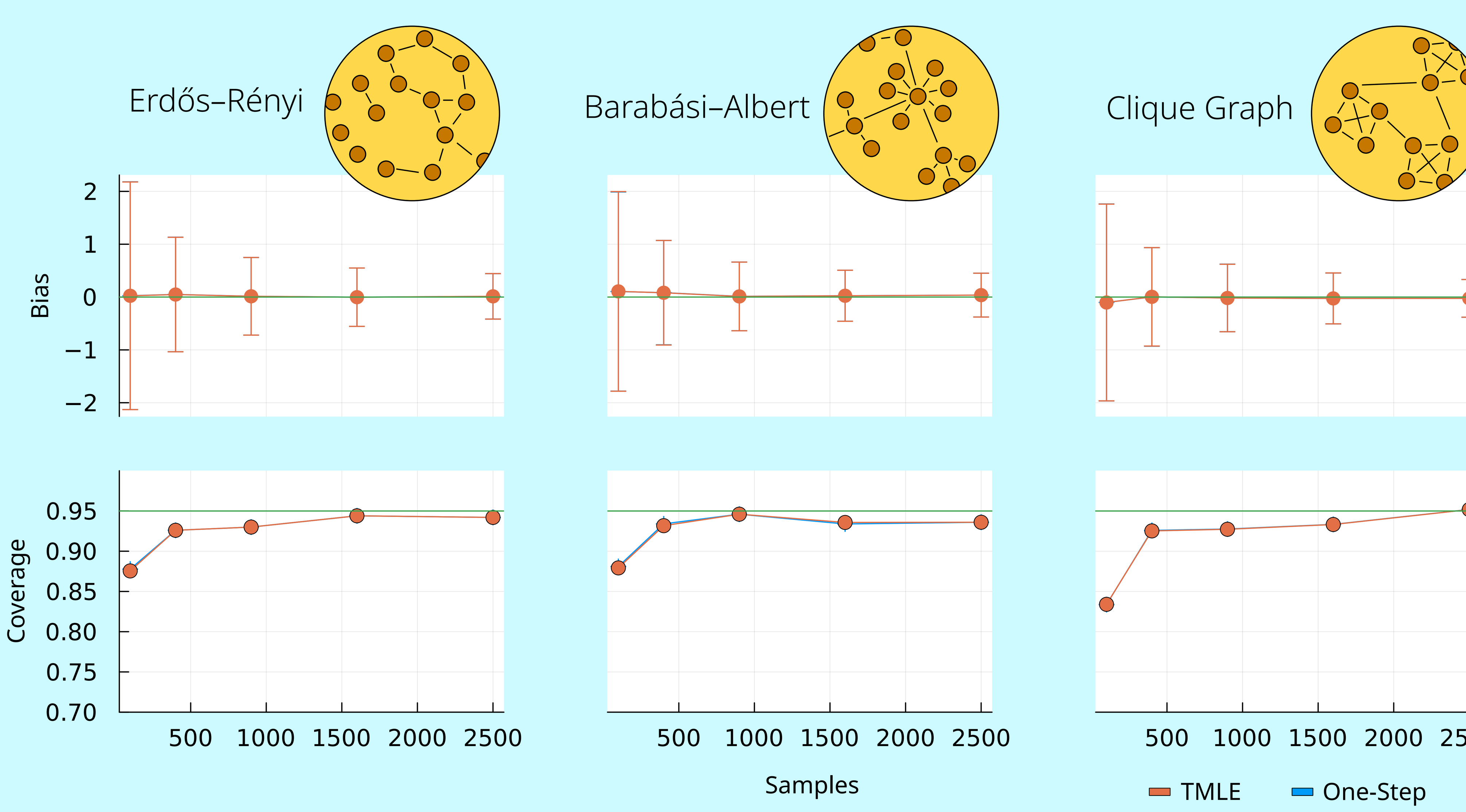
One-Step: $\hat{\psi}_n = \frac{1}{n} \sum_{i=1}^n H_i(Y_i - \bar{Q}_i(a_i^s, w_i^s)) + \bar{Q}_i(h(a_i^s; w_i^s, \delta))$

TMLE: $\hat{\psi}_n = \frac{1}{n} \sum_{i=1}^n \bar{Q}_i^\epsilon$ where $\bar{Q}_i^\epsilon$ fit via submodel fluctuation

Semiparametric Efficient & Doubly Robust Consistent

Simulations

- Evaluated Sum and Additive Shift with outcome a combination of nonlinear functions of 8 covariates + sine function treatment.
- Estimators: TMLE and One-Step with Super Learning for conditional mean, KLIEP for density ratio.
- 500 iterations for each sample size; EIF-based variance estimator for correlated treatment from Ogburn et al. (2022).



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